Comments on Le Maohua's 1999 paper in the Proc. Japan Acad.

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MSC: 11D61, 11D99

Considering the equation

$$a^x + b^y = c^z \tag{1}$$

where a, b, c are coprime, squarefree positive integers such that c odd, [Le] gave $2^{\omega(c)+1}$ as an upper bound on the number of solutions (x, y, z), where $\omega(c)$ is the number of distinct prime factors of c. He also showed that

$$z < 2ab\log(2eab)/\pi \tag{2}$$

for any solution (x, y, z) to (1). Here we give slight improvements to each of these results, also removing the restriction that a, b, c be squarefree:

Theorem For positive integers a, b, c, with c odd, (1) has at most $2^{\omega(c)}$ solutions (x, y, z), where $\omega(c)$ is the number of distinct prime factors of c. All solutions (x, y, z) to (1) satisfy $z < ab/2$.

Proof: The first assertion follows from the fact that, of the four parity possibilities for the pair (x, y), only two are possible in (1): this follows from the proof of Theorem 6 of [Sc]. The second assertion follows from Theorem 3 of [Sc-St], noting that $n > n^{1/2}\log(n)$ for $n \geq 2$.

References

[Le] M. Le, An upper bound for the number of solutions of the exponential diophantine equation $a^x + b^y = c^z$, Proc. Japan Acad., **75**, Ser. A (1999)

[Sc] R. Scott, On the Equations $p^x - b^y = c$ and $a^x + b^y = c^z$, *Journal of Number Theory*, **44**, no. 2 (1993), 153-165.

[Sc-St] R Scott and R. Styer, On $p^x - q^y = c$ and related three term exponential Diophantine equations with prime bases, *Journal of Number Theory*, **105** no. 2 (2004), 212–234.